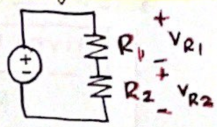
Basics: Resistors and capacitors

	Resistors	Capacitors
General Construction	$R = \rho \frac{\text{length}}{\text{Area}}$	$C = K\varepsilon_0 \frac{\text{Area}}{\text{distance}}$
I-V relation	$V = IR$	$Q = CV \rightarrow I = C\frac{dV}{dt}$
series equivalence	$R_{eq} = R_1 + R_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ } $C_1 \parallel C_2 = \frac{C_1 C_2}{C_1 + C_2}$
parallel	opposite →	↖ opposite
energy stored	resistors don't store energy	$E = \frac{1}{2}CV^2$

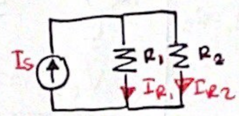
Voltage Divider in series



$V_{R1} = V_S \frac{R_1}{R_1 + R_2}$

$V_{R2} = V_S \frac{R_2}{R_1 + R_2}$

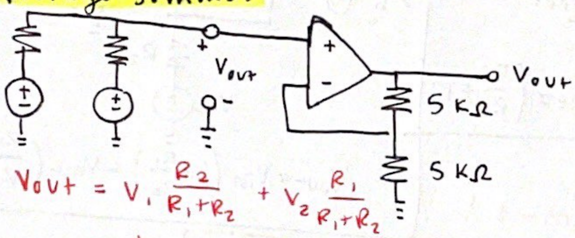
Current Divider in ~~Series~~ Parallel



$I_{R1} = I_S \frac{R_2}{R_1 + R_2}$

$I_{R2} = I_S \frac{R_1}{R_1 + R_2}$

Voltage summer



$V_{out} = V_1 \frac{R_2}{R_1 + R_2} + V_2 \frac{R_1}{R_1 + R_2}$

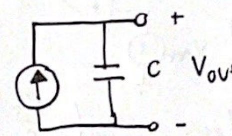
$= \frac{1}{2}V_1 + \frac{1}{2}V_2$

*if $R_1 = R_2$

so $V_{out} = V_1 + V_2$

Capacitors (Basic circuits)

* use $I = C\frac{dV}{dt}$!



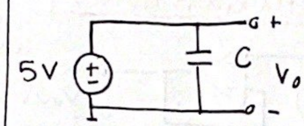
$Q = CV$

$\frac{d}{dt}Q = \frac{d}{dt}CV$

$I = C\frac{dV}{dt}$

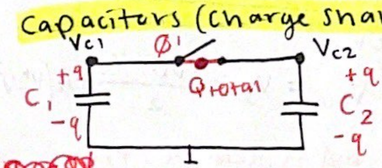
$\therefore V_{out} = \frac{I}{C}t + V_0$

initial charge of capacitor



$Q = CV$!

Capacitors (charge sharing)



$C_1 = C_2 = 1\mu F$

once ϕ_1 closes:

$V_{c1} = 1V = 1V$ } currently
$V_{c2} = 2V$

$Q_{tot} = +q_{c1} + q_{c2}$

$C_1: q_{c1}^{\phi_1} = C_1 V_{c1} = (1\mu F)(1V) = 1\mu C$

$C_2: q_{c2}^{\phi_1} = C_2 V_{c2} = (1\mu F)(2V) = 2\mu C$

} $Q_{tot} = 3\mu C$

$$V_f = \frac{C_1 V_{c1} + C_2 V_{c2}}{C_1 + C_2}$$

← general formula!

find V final:

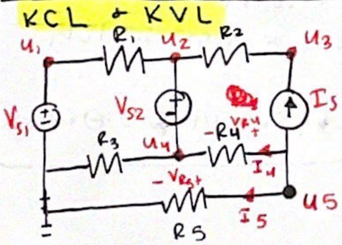
$C_1 V_f + C_2 V_f = 3\mu C$

$V_f(C_1 + C_2) = 3\mu C$

$V_f = \frac{3}{2}V$ — charge on both caps

$Q = CV \rightarrow Q = 1\mu F(\frac{3}{2}) = \frac{3}{2}\mu C$

KCL + KVL



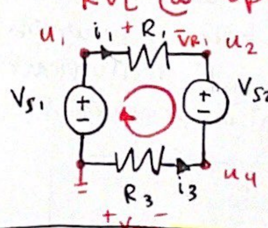
KCL @ u_5

$I_{in} = I_{out}$

$0 = I_s + i_4 + i_5$

$$0 = I_s + \frac{u_5 - u_4}{R_4} + \frac{u_5}{R_5}$$

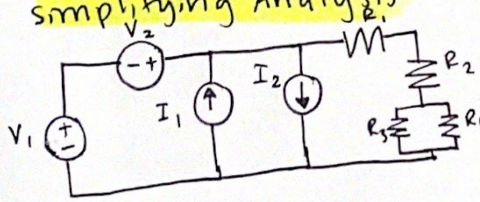
KVL @ upper left

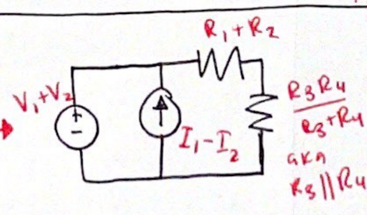


$-V_{S1} + V_{R1} + V_{S2} + V_{R3} = 0$

$-V_{S1} + (u_1 - u_2) + V_{S2} + u_4 = 0$

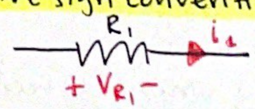
Simplifying Analysis





$\frac{R_3 R_4}{R_3 + R_4}$ aka $R_3 \parallel R_4$

Passive sign convention



* current flows + → -

Power

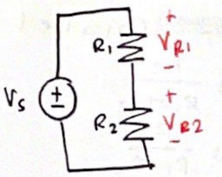
$P = IV$ Always!

* only resistors can use $P = \frac{V^2}{R}$ + $P = I^2 R$

* power is always conserved! sum of all power = 0!

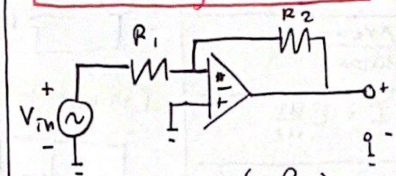
REFERENCE CIRCUITS

Voltage Divider



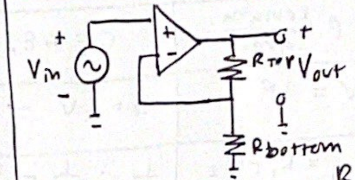
$$V_{R_2} = V_s \frac{R_2}{R_1+R_2}$$

Inverting Op-Amp



$$V_{out} = V_{in}\left(-\frac{R_2}{R_1}\right)$$

Non-Inverting Op-Amp

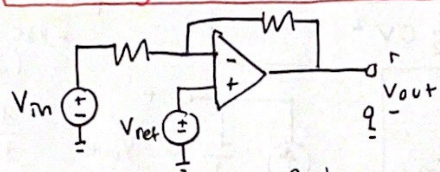


$$V_{out} = V_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right)$$

Current Divider

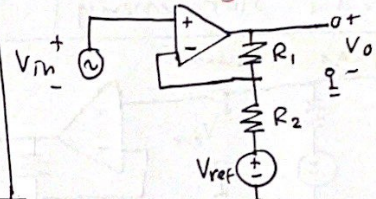
$$I_1 = I_s\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amp w/ Reference



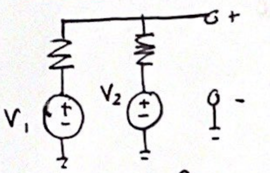
$$V_{out} = V_{in}\left(-\frac{R_f}{R_s}\right) + V_{REF}\left(\frac{R_f}{R_s}+1\right)$$

Non-Inverting Amp w/ Ref



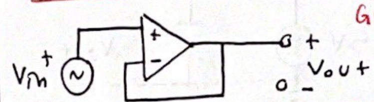
$$V_{out} = V_{in}\left(1+\frac{R_1}{R_2}\right) - V_{ref}\left(\frac{R_1}{R_2}\right)$$

Voltage Summer



$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



Gain = 1 !

$$V_{out} = V_{in}$$

*allows it to give a clearer undistorted signal

Gain

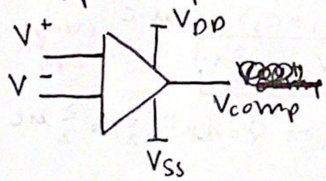
if Gain > 0 → non-inverting

if Gain < 0 → inverting!

Op Amps

→ comparators: compares 2 voltages

→ Op-Amp: operational Amplifier

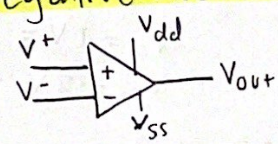


if $V^+ > V_-$; $V_{comp} = V_{DD}$

if $V^+ < V_-$; $V_{comp} = V_{SS}$

- amplifies signals
- isolate circuits to added loading effect
  - ↳ "loading effect" = degree to which the measurement instrument impacts electrical properties of the circuit.

Negative Feedback



$$V_{out} = V_{ss} + \frac{V_{DD}+V_{ss}}{2} + A(V^+ - V^-)$$

* A is ideal! $A = +\infty$

for an op-amp in negative feedback:

$V_{in} - f \cdot V_{out} = V_{err}$ (↑ = 0) ∴ $V_{out} = \frac{A}{1+Af} V_{in}$

Checking for negative feedback

① zero out all indep sources
- ↳ voltage source = wire
- ↳ current sources = open switch

② wiggle!
- ↳ if error signal ↓, output ↓ = ⊖ fdbck

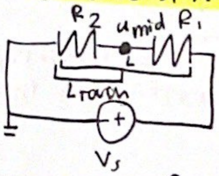
Golden Rules

① $I_+ = I_- = 0$

② $u_+ = u_-$
- ↳ only when in ⊖ feedback
- ↳ $A = +\infty$
- ↳ $V_{error} = 0$ aka $u_+ - u_-$
- ↳ $V_{out} = A\,V_{error} = A(u_+ - u_-)$

$$V_{out} = \frac{A}{1+Af}u^+ \rightarrow u^- = fV_{out} = \frac{fA}{1+Af}u^+$$

1D Resistive Touchscreen



$$R_2 = \rho \frac{L_{touch}}{A}$$

$$R_1 = \rho \frac{L_{rest}}{A}$$

$$V_{out} = \frac{R_2}{R_1 + R_2} V_s$$

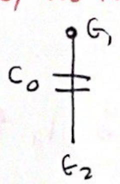
*umid is the touch!

$$u_{mid} = \frac{R_2}{R_1 + R_2} V_s = \frac{L_{touch}}{L} V_s$$

Superposition

- voltage source = wire
- current source = open switch
- find the value of an element in each "circuit"
- add together the current + voltage

w/ no finger



w/ finger equations

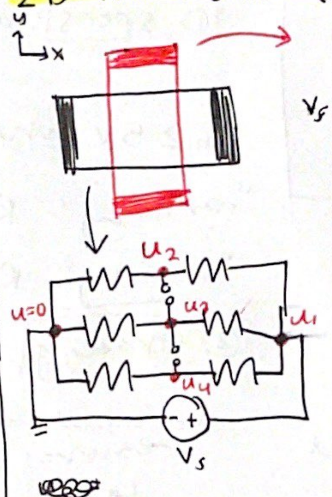
$$C_0 = \varepsilon \frac{d_2 w_1}{t_1}$$

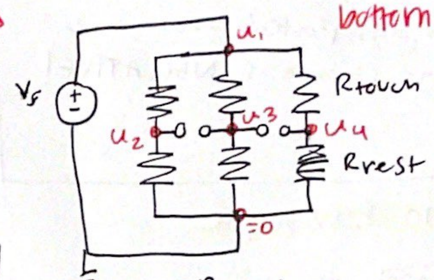
$$C_{F-E_1} = \varepsilon \frac{d_1 w_1}{t_2 - t_1}$$

$$C_{F-E2} = \varepsilon \frac{d_2 (w_2 - w_1)}{t_2}$$

2D Touchscreen (Resistive)

*top = y-coord
bottom = x-coord



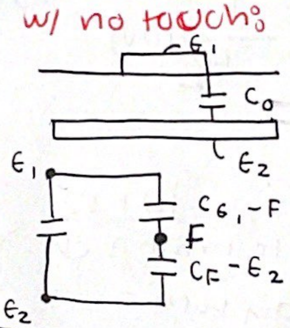


$$u_3 = \frac{R_{touch}}{R_{rest} + R_{touch}} V_s$$

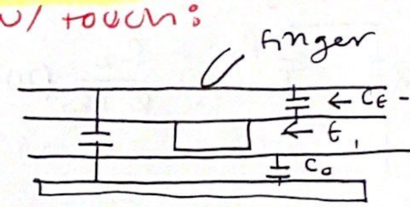
aka $u_3 = \frac{L_{touch}}{L} V_s$ ← vertical for top! / horizontal for bottom

Capacitative Touchscreen

w/ no touch:

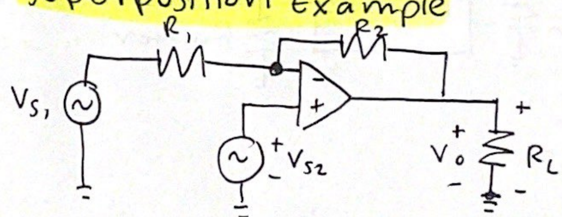


w/ touch:



$$(C_{E \parallel} = \parallel C_{EF-E_2}) + C_0$$

Superposition Example



$$V_0 = \left(1 + \frac{R_2}{R_1}\right) V_{S2} - \left(\frac{R_2}{R_1}\right) V_{S1}$$

V_{S2} was a non-inverting op amp

V_{S1} is inverting!

Random Reminders

- power dissipated from the voltage source is NEGATIVE!

Design Example

Problem

$V_m > 0 \rightarrow$ forward

$V_m < 0 \rightarrow$ backward

$|V_m|$ is proportional to speed

① specs: distance ↓, speed ↓

we want decreasing positive voltage for this

$V_m \geq 5V$ when "far away"

"far away" $R_{photosensor} = 10 K\Omega$

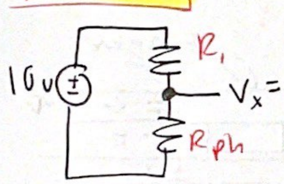
"nearby" $R_{photosensor} = 100\Omega$

② strategy as D↑, L↓, $R_{photosensor}$↑

we need to build something that measures resistance → output voltage!

↳ try voltage divider!

③ solve



$$V_x = \frac{R_2}{R_1 + R_2}(10V) = \frac{1}{1 + R_1/R_2}(10V)$$

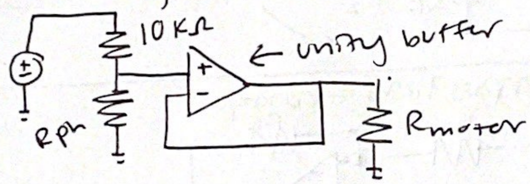
as R_1↑, $\left(\frac{R_1}{R_2}\right)$↑, V_x↓

R_2↑, $\left(\frac{R_1}{R_2}\right)$↓, V_x↑

make $R_2 = R_{ph}$

* need $V_x = 5V$ when $R_{ph} = 10 k\Omega$

use a unity buffer! to isolate it :)



D↑, L↓, R_{ph}↑, $\left(\frac{R_1}{R_{ph}}\right)$↓, V_x↑ ✓